WIPL-D CMA Solver

Characteristic Modes Analysis in WIPL-D Software Package

Introduction

- Characteristic mode analysis (CMA) is the numerical calculation of a weighted set of orthogonal current modes that are supported on a given structure
- Characteristic modes depend only on the physical parameters of the structure
  - They do not depend on the way the structure is excited
- Calculation of characteristic modes provide a deep insight into the physical behavior of the analyzed structure
  - This helps designers to follow a systematic, intelligent approach rather than a brute force method, by looking at the eigenvalue spectrum and the eigenvector distribution

EFIE formulation

- EFIE formulation for the perfectly conductive body

$$\boldsymbol{L(J)}+\boldsymbol{E^{i}}=0$$

Where $\boldsymbol{J}$ is current over the body surfaces, while $\boldsymbol{E^{i}}$ is excitation electric field

- Operator $\boldsymbol{L}$ builds the relationship between scattered electric fields and surface currents
- For tangential component we can write:

$$\left[\boldsymbol{L(J)}+\boldsymbol{E^{i}}\right]_{tan}=0$$

- If we write tangential component of operator $\boldsymbol{L}$ as a new operator $\boldsymbol{Z}$, we have:

$$[\boldsymbol{L}(\boldsymbol{J})]_{tan}=\boldsymbol{Z}(\boldsymbol{J}) \quad \rightarrow \quad \boldsymbol{Z}(\boldsymbol{J})=-\left(\boldsymbol{E^{i}}\right)_{tan}$$

Eigenvalue Equation

- Operator ***Z*** has the dimension of impedance, and it can split into its real and imaginary part:

$$\boldsymbol{Z}=\boldsymbol{R}+j\boldsymbol{X}$$

- Harrington and Mautz introduced eigenvalue equation as:

$$\boldsymbol{Z}(\boldsymbol{J_n}) = v_n \boldsymbol{R}(\boldsymbol{J_n})$$

where J_n is eigenvector and v_n is eigenvalue

- By combining two equations written above, we obtain eigenvalue equation given by:

$$\boldsymbol{X}(\boldsymbol{J_n}) = \lambda_n \boldsymbol{R}(\boldsymbol{J_n}), \text{ with } \quad v_n = 1 + j\lambda_n$$

- It is common practice to refer to λ_n as the eigenvalues, rather than

Physical Interpretation of Eigenvalues

- Eigenvalues (λ_n) are real numbers, within the range $[-\infty, +\infty]$
- Total energy stored by an eigenmode is given by:

$$\omega \iiint_v (\mu H_n \cdot H_n{}^* - \varepsilon E_n \cdot E_n{}^*) dv = \lambda_n$$

- Total stored field energy is proportional to the magnitude of the eigenvalues
  - Eigenmodes with λ_n<0 store electric energy - capacitive modes
  - Eigenmodes with λ_n>0 store magnetic energy - inductive modes
  - Eigenmodes with λ_n=0 are resonant modes, and they radiate most efficiency

Modal Significance

- Modal significance is defined as:

$$\mathrm{MS} = \left|\frac{1}{1+j\lambda_n}\right|$$

- Modal significance is positive real number, with values in the range between 0 and 1
  - Much easier to inspect modal significances graph (range 0 to 1) than eigenvalue graph (values -∞ to +∞)

- Modes with significance close to 0 do not significantly contribute to radiation, while the modes with modal significance close to 1 do
  - Mode with modal significance equal to 1 is resonant mode

Characteristic Angle

- Characteristic angle is defined as:

$$\alpha_n = 180° - \operatorname{atan}(\lambda_n)$$

- Physically, characteristic angle is phase difference between modal current $\boldsymbol{J}_n$ and electric field produced by this current $\boldsymbol{E}_n^{tan}$

- Starting from physical interpretation of eigenvalue, it is obvious that:
  - $\alpha_n = 180°$, the corresponding mode is resonant mode
  - $90° < \alpha_n < 180^0$, the corresponding mode is inductive mode
  - $180° < \alpha_n < 270^0$, the corresponding mode is capacitive mode

Orthogonality of Characteristic Modes

- After the eigenvalue equation is solved, eigenvectors (characteristic currents) are usually normalized to unit radiation power

$$\langle \boldsymbol{R} \cdot \boldsymbol{J}, \boldsymbol{J}^{*} \rangle = 1$$

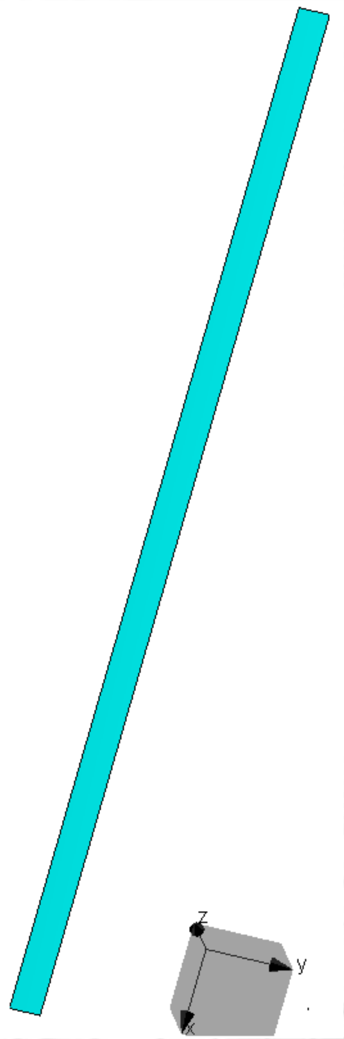
- Larger electric currents in high-order modes (small modal significance) are required to achieve unit radiation power than in low-order modes (high modal significance)
- Orthogonality properties of the normalized characteristic currents are given by:

$$\langle \boldsymbol{R} \cdot \boldsymbol{J}_m, \boldsymbol{J}_n{}^{*} \rangle = \delta_{mn}$$
$$\langle \boldsymbol{X} \cdot \boldsymbol{J}_m, \boldsymbol{J}_n{}^{*} \rangle = \lambda_n \delta_{mn}$$
$$\langle \boldsymbol{Z} \cdot \boldsymbol{J}_m, \boldsymbol{J}_n{}^{*} \rangle = (1 + j\lambda_n) \delta_{mn}$$

- Characteristic fields (far-fields due to characteristic currents) also have orthogonality properties

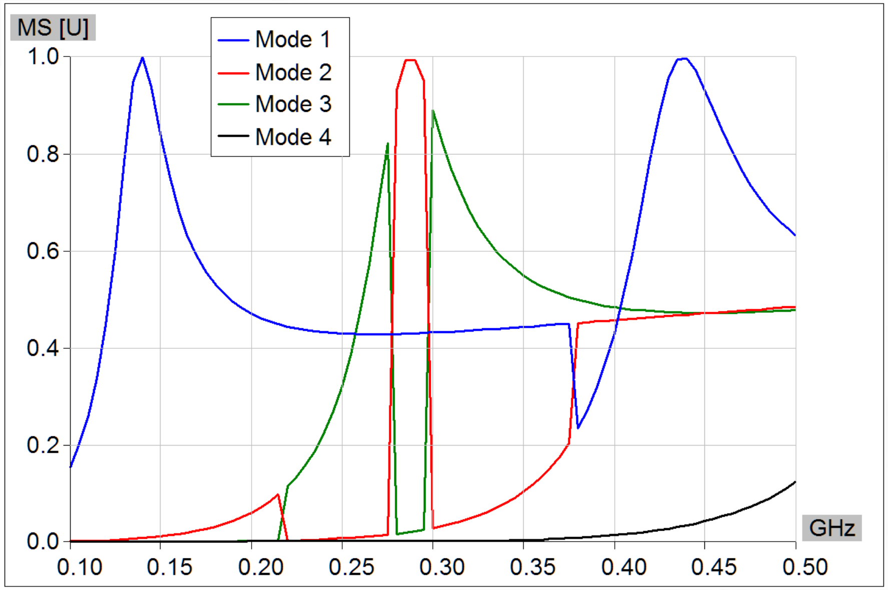
CMA of Narrow Metallic Plate

- Infinitely thin metallic plate is analyzed
  - Dimensions: 100cm x 3cm
  - Frequency range: 100 to 500 MHz
- WIPL-D kernel fill MoM matrix, with only 16 unknowns
  - Total number of eigenvalues and eigenvectors is 16
  - Characteristic mode analysis is performed for the first 4 modes
- Key steps:
  - Calculate of MoM system matrix
  - Build the eigenvalue equation
  - Solve the eigenvalue equation
  - Calculate characteristic currents

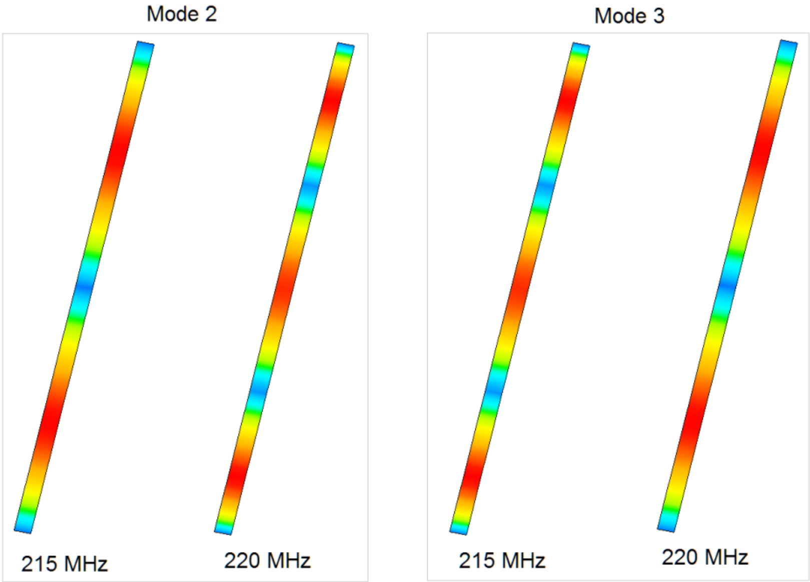


Solution of the Eigenvalue Equation

- Modal Significance for the first 4 modes



- Modal currents for the second and the third mode at 215MHz and 220MHz



- Modes with the same index number at different frequencies do not automatically refer to the same physical modes

Mode Tracking

- Mode Tracking is required in order to provide that the modes with the same index number at different frequencies refer to the same physical modes
  - The same mode refers to the mode that has similar current distribution and modal field patterns

- Algorithm for mode tracking which is implemented in WIPL-D solver is based on utilization of orthogonality of modal currents

Mode Tracking - Algorithm

- Mode Tracking algorithm consists of 3 major steps
  - Normalization of all modal currents at each analyzed frequency
  - Computation of correlation coefficients between modes at two adjacent frequencies, according to the following equations

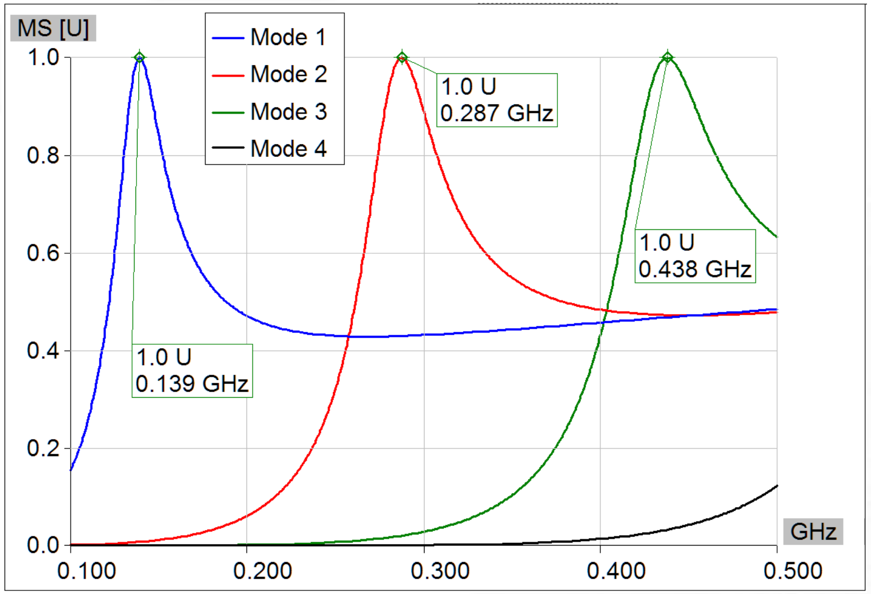
$$c_{ij} = \left|\rho\left(\boldsymbol{J}_i(\omega_1), \boldsymbol{J}_j(\omega_2)\right)\right|$$

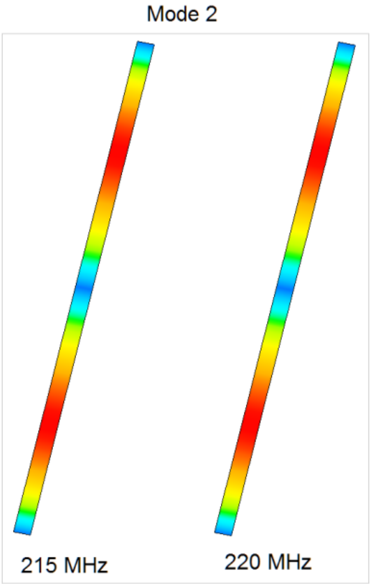
$$\rho(\boldsymbol{x}, \boldsymbol{y}) = \frac{N\sum_{i=1}^{N} x_i y_i - \sum_{i=1}^{N} x_i \cdot \sum_{i=1}^{N} y_i}{\sqrt{N\sum_{i=1}^{N} {x_i}^2 - \left(\sum_{i=1}^{N} x_i\right)^2} \cdot \sqrt{N\sum_{i=1}^{N} {y_i}^2 - \left(\sum_{i=1}^{N} y_i\right)^2}}$$

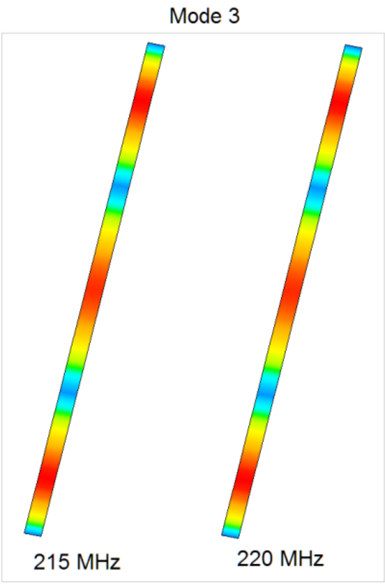
  - Association of the eigenvectors at two frequencies iteratively, based on values of correlation coefficients

Results with Mode Tracking

- Modal Significances, and currents for 2nd and 3rd modes at 215MHz and 220MHz, after the mode tracking is applied



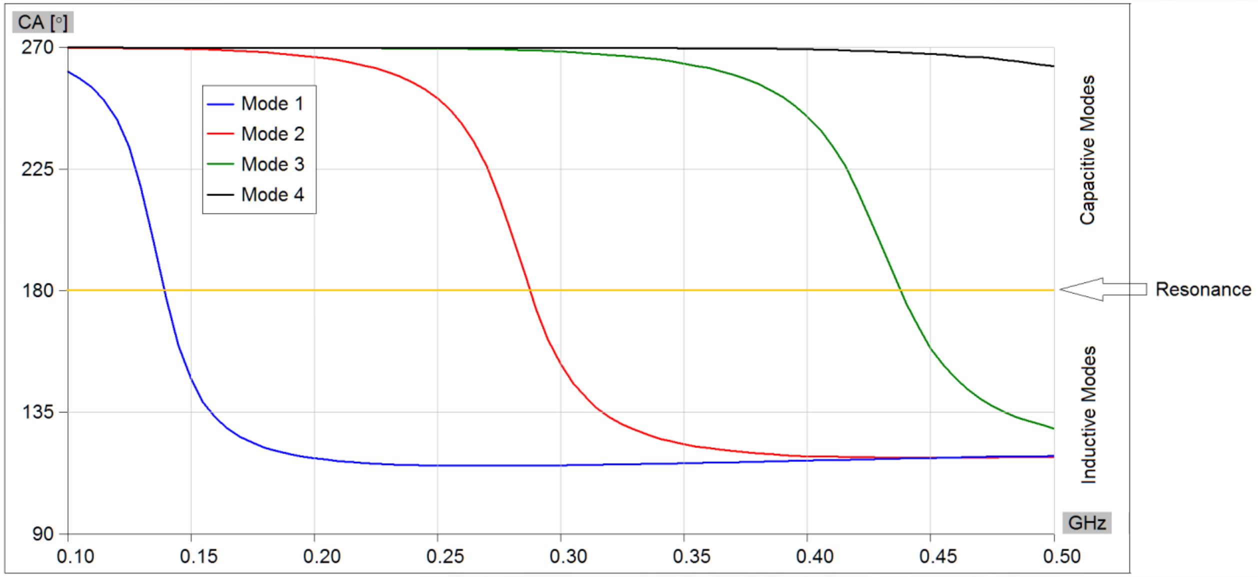




- Resonances are at expected frequencies, where length of the plate is equal to $n \cdot \frac{\lambda}{2}$

Characteristic Angle

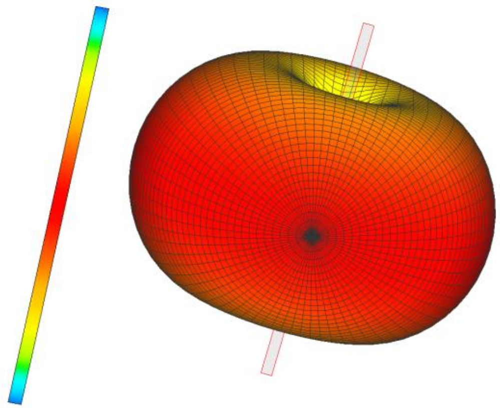
- Characteristic Angle for the first 4 modes



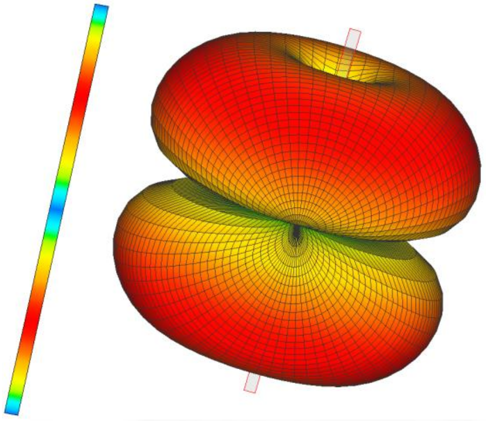
- Theoretically expected results
  - Capacitive behavior of electrically short antennas
  - Inductive behavior of electrically long antennas
  - Resonance for antenna length equal to $n \cdot \frac{\lambda}{2}$

Comparison to dipole antenna

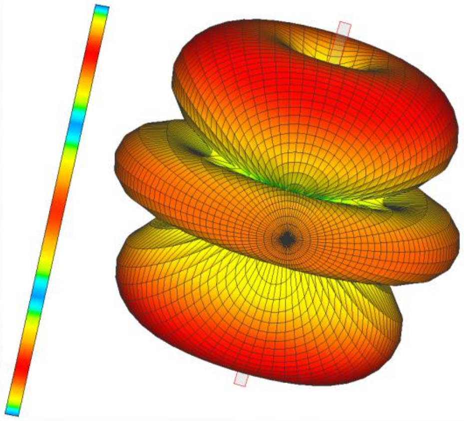
- Current distribution and Radiation Pattern of resonant modes



Mode 1

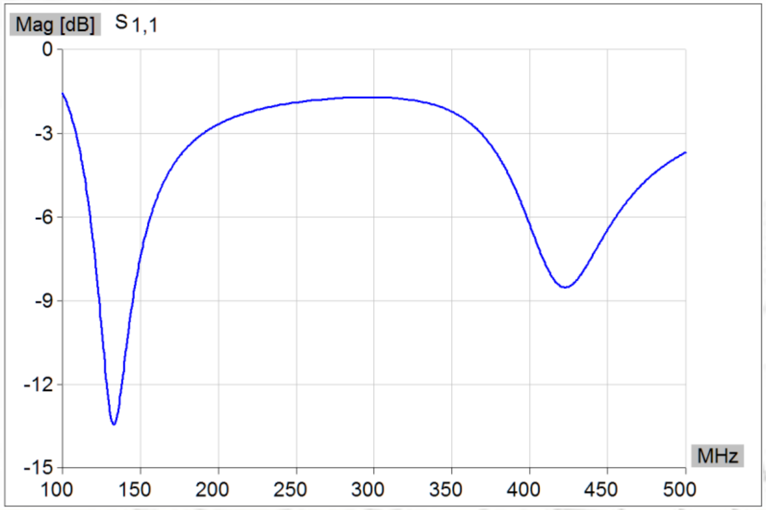


Mode 2



Mode 3

- Dipole antenna fed in its central point
  - Resonances corresponds to resonances of modes 1 and 3
  - Mode 2 is not excited as antenna is centrally fed



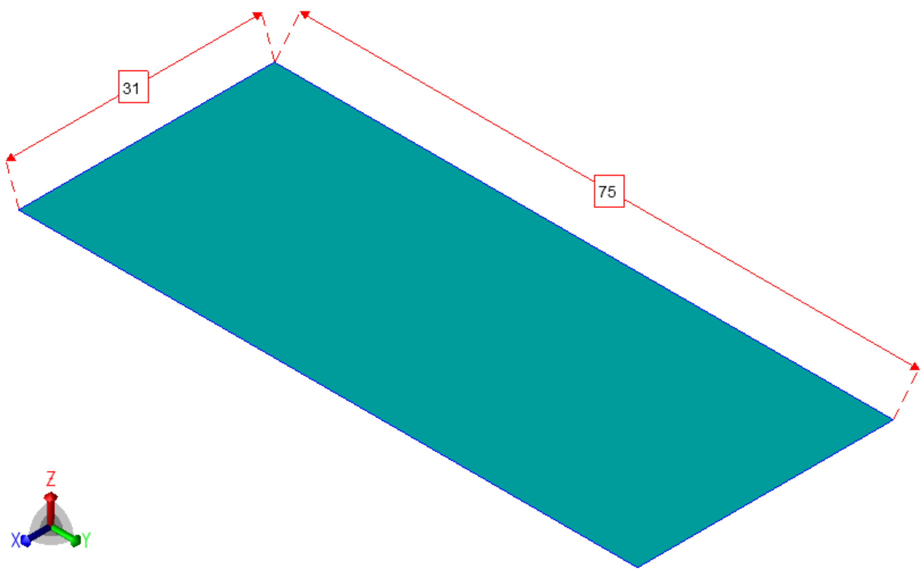
Antenna Design based on CM Analysis

- Calculation of characteristic modes provide a clear understanding of physical behavior of the analyzed structure
  - The inspection of the natural current flows and their accompanying radiation patterns helps determine which mode(s) to excite and where to place the sources in order to obtain desired antenna performances

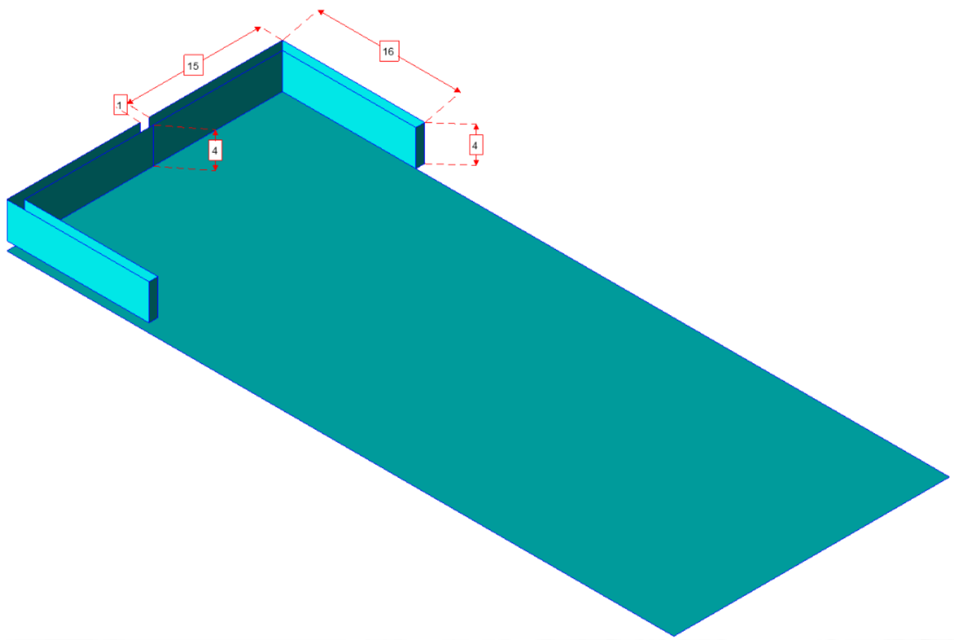
- Design of MIMO antennas is often based on CMA
- Basic idea comes from the orthogonality of characteristic modes currents and fields:
  - If ports of MIMO antenna excites different modes (or set of modes) MIMO antenna will have very small port-to-port coupling

CMA of a Folded Dipole Model

- CMA analysis of folded dipole and corresponding ground plane
  - Frequency range: 1 GHz to 5 GHz



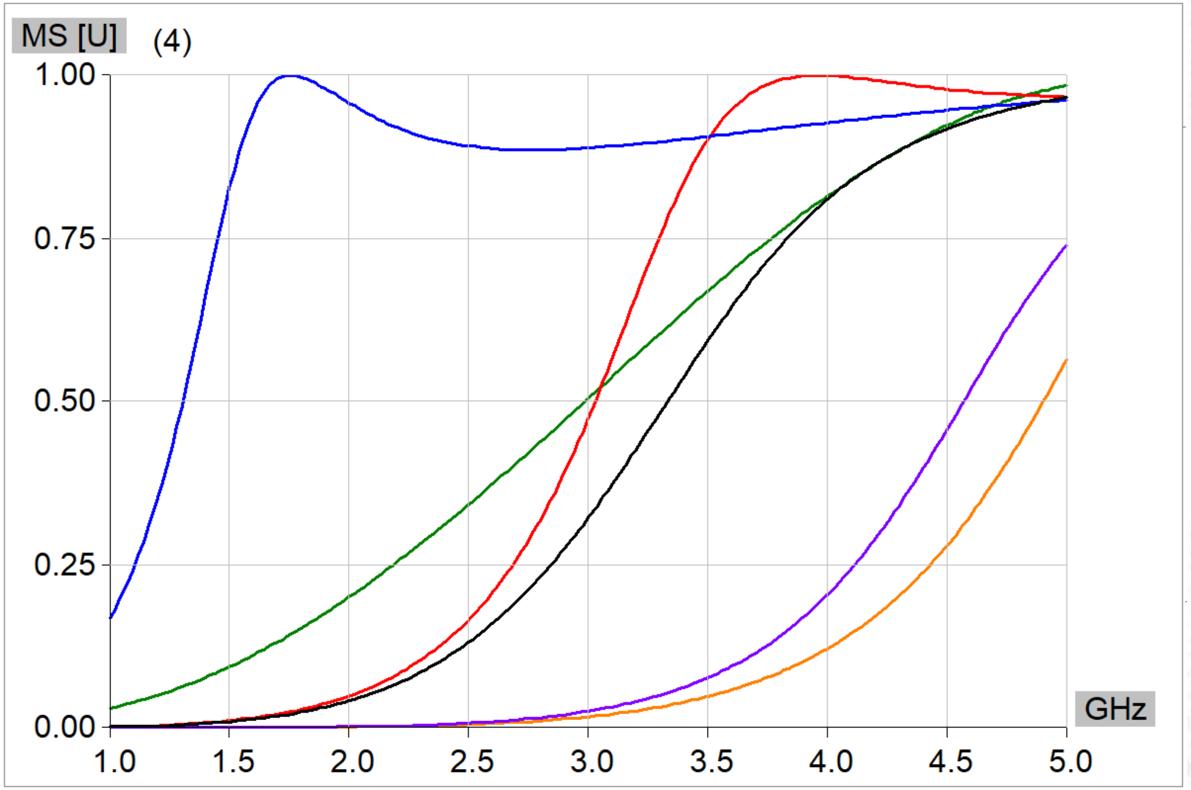
Ground



Folded dipole

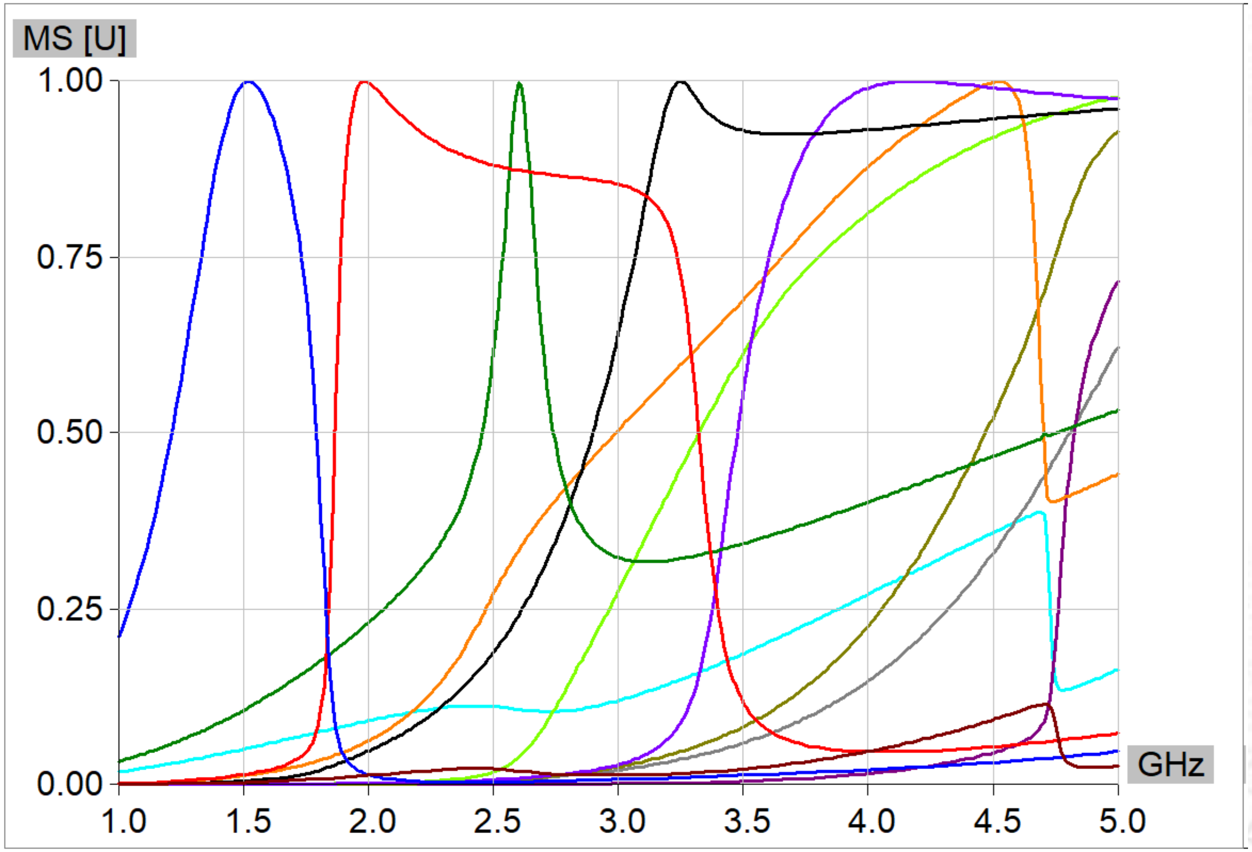
Modal Significances for Ground Plane

- Modal significance of the ground plane (75x31 mm in size)
  - 271 unknowns
  - Modal significance for the first 6 modes presented



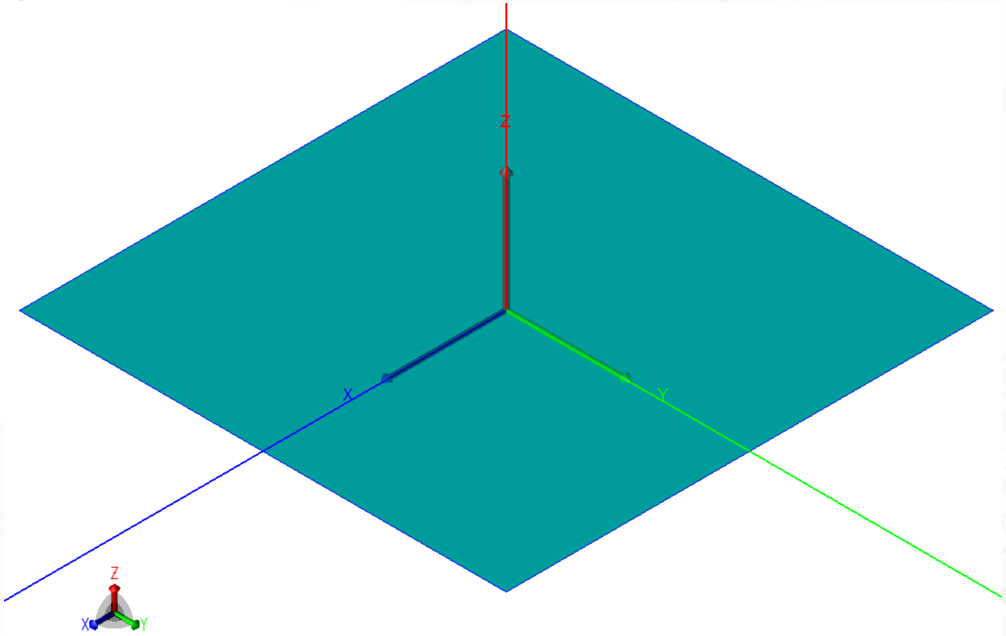
Modal Significances for Folded Dipole

- Modal significance for the first 15 modes presented
  - Model simulated in 351 frequency points
  - 521 unknowns



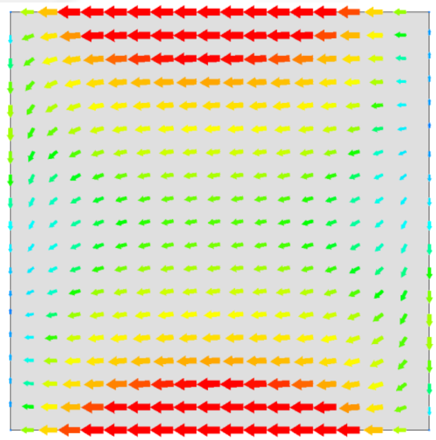
MIMO Antenna Design – Initial CMA

- We start from the CM Analysis of an infinitely thin perfectly conductive plate
  - Plate dimensions are $150 \times 150mm$
  - Frequency of interest is 1 GHz
  - Modes of interest are the first 6 modes (6 modes with the highest Modal Significance)

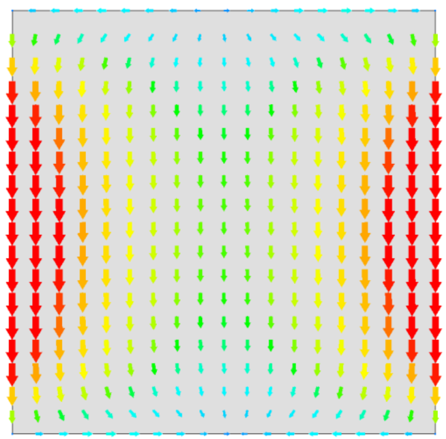


MIMO Antenna Design – Characteristic Currents

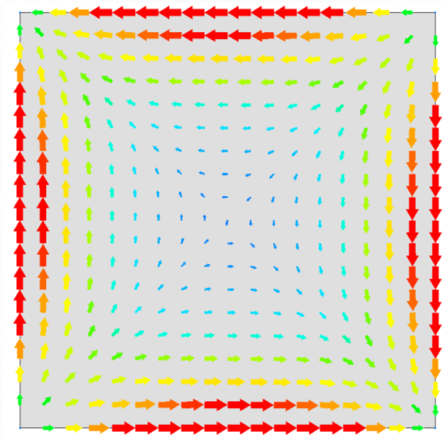
- Characteristic Currents of the first 6 modes



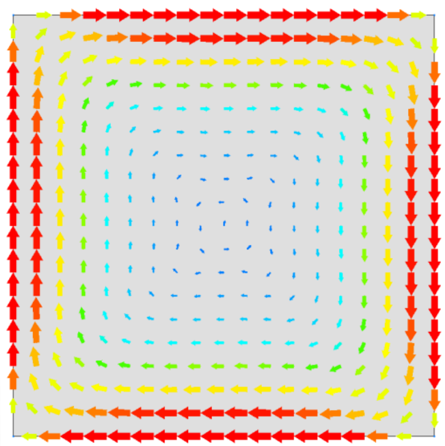
Mode 1



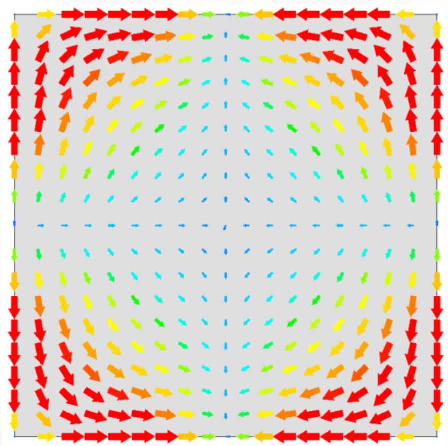
Mode 2



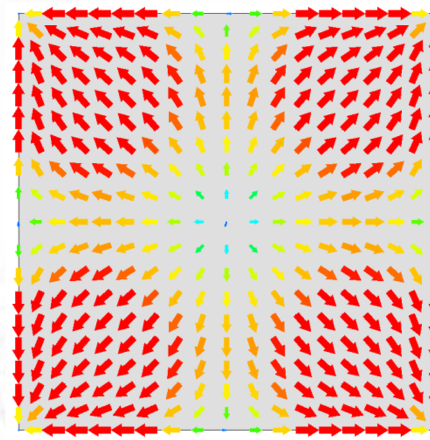
Mode 3



Mode 4



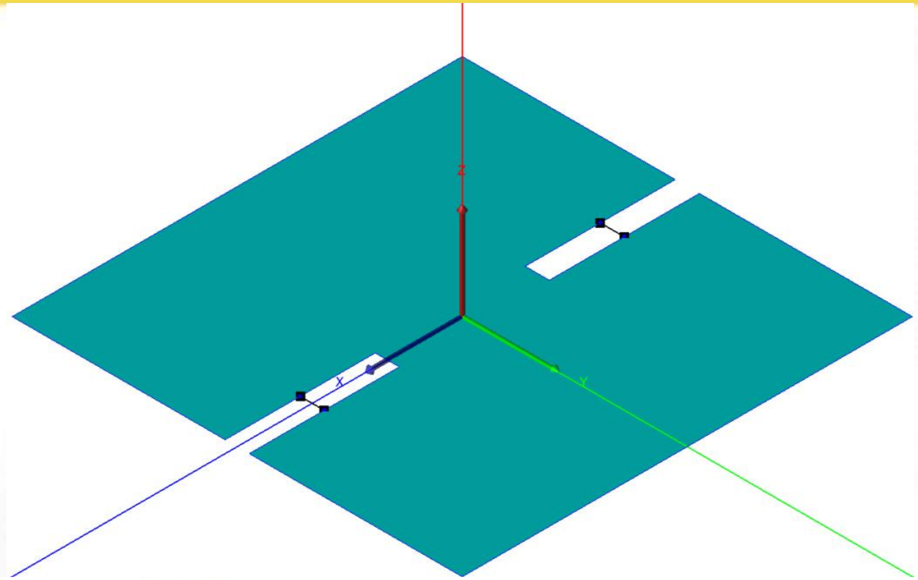
Mode 5



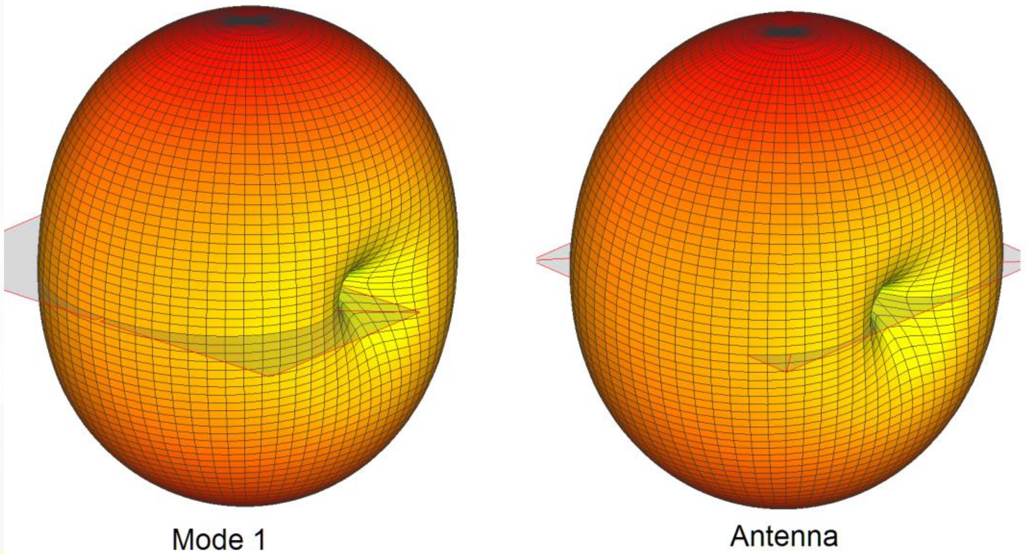
Mode 6

MIMO Antenna Design – Exciting of the Mode 1

- Two slots are made on the centers of the opposing plate edges
  - Generators with the equal voltages placed on the both slots

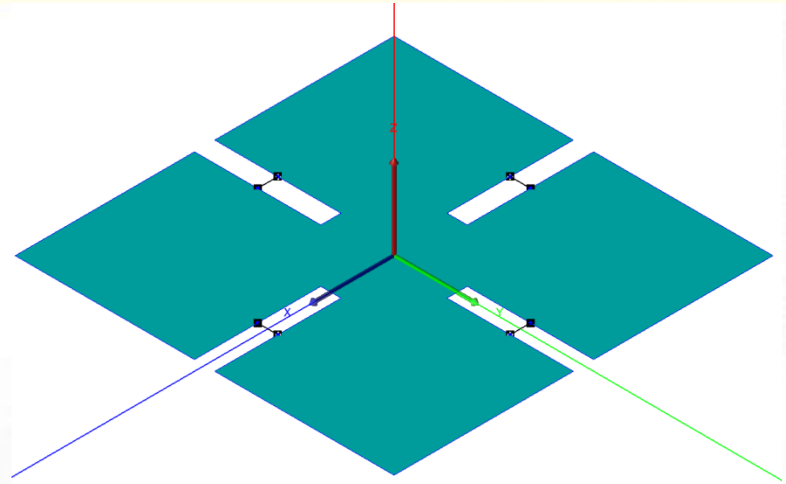


- Comparison of the Radiation pattern of the Mode 1 and the antenna



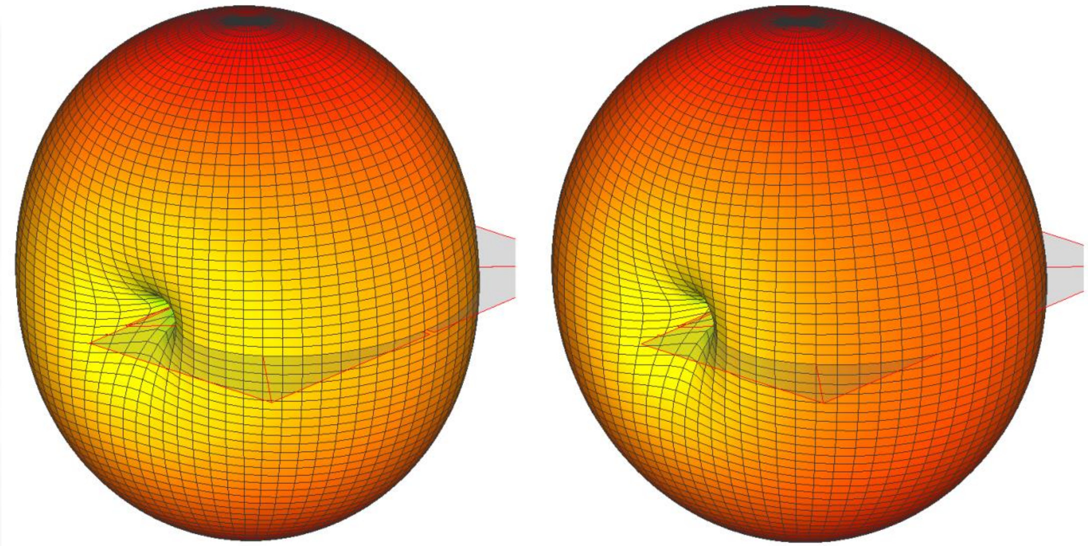
MIMO Antenna Design – Exciting of the Mode 2

- Two slots are made and excited in the exactly same way as for the Mode 1, but entire constellation is rotated for 90 degrees versus the plate axis (z-axis)



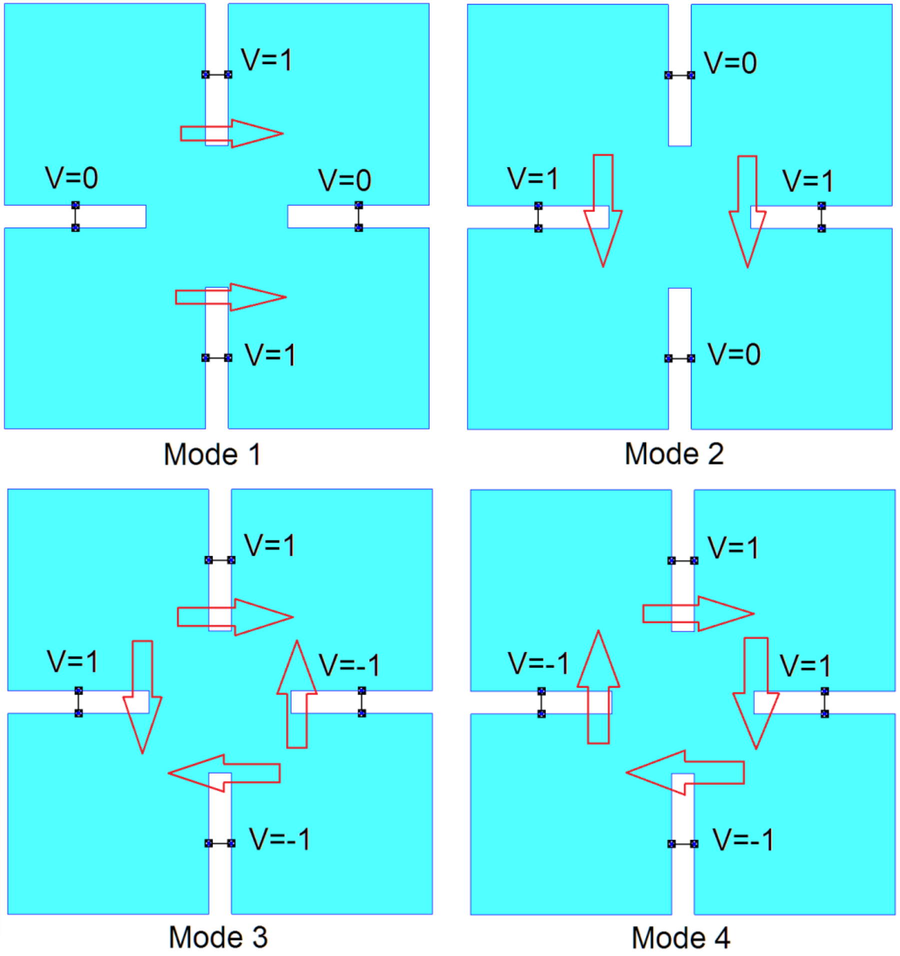
2-modes antenna

- Comparison of the Radiation pattern of the Mode 2 and the antenna, with 3rd and 4th slots excited



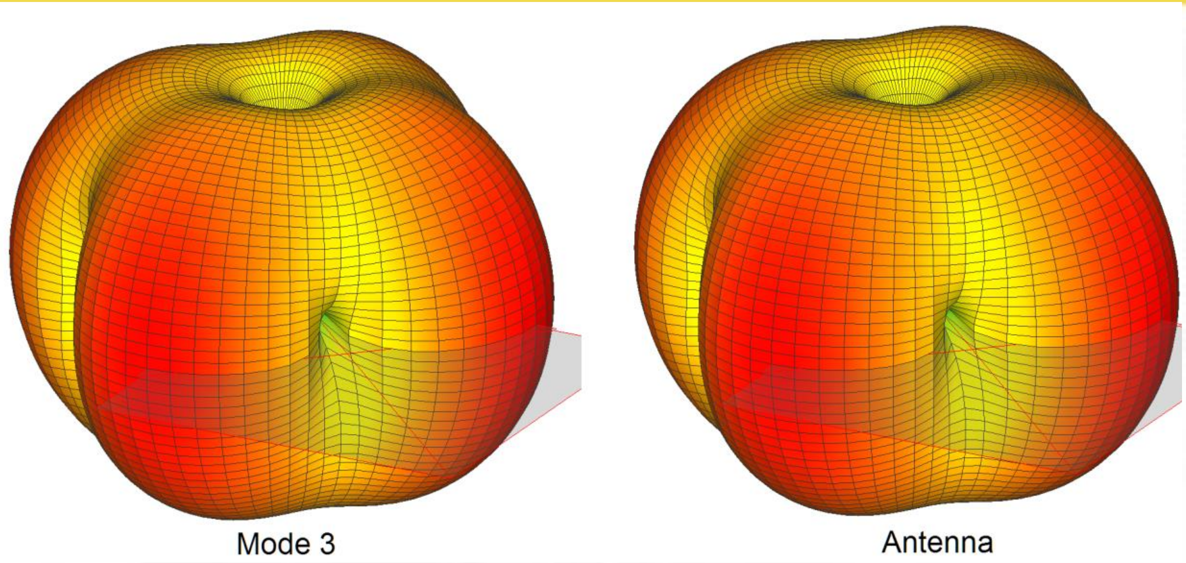
MIMO Antenna Design – Exciting of the Modes 1 to 4

- Antenna configuration used for modes 1 and 2
  - Generators voltages used for exciting of modes 1, 2, 3 and 4

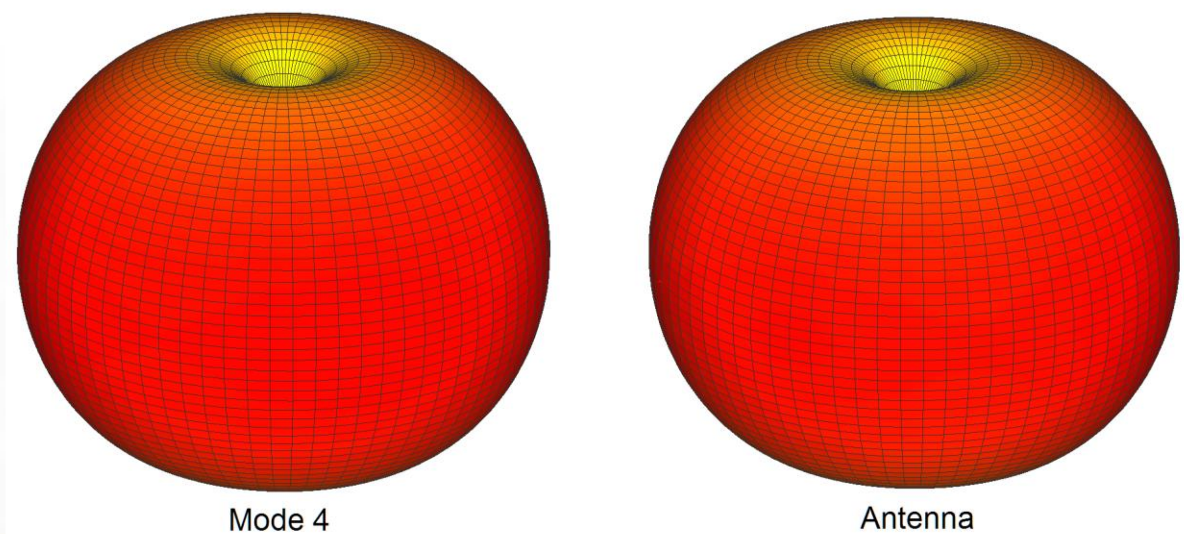


MIMO Antenna Design – Radiation Patterns of modes 3 and 4

- Comparison of the radiation pattern of Mode 3 and antenna, excited to produce Mode 3

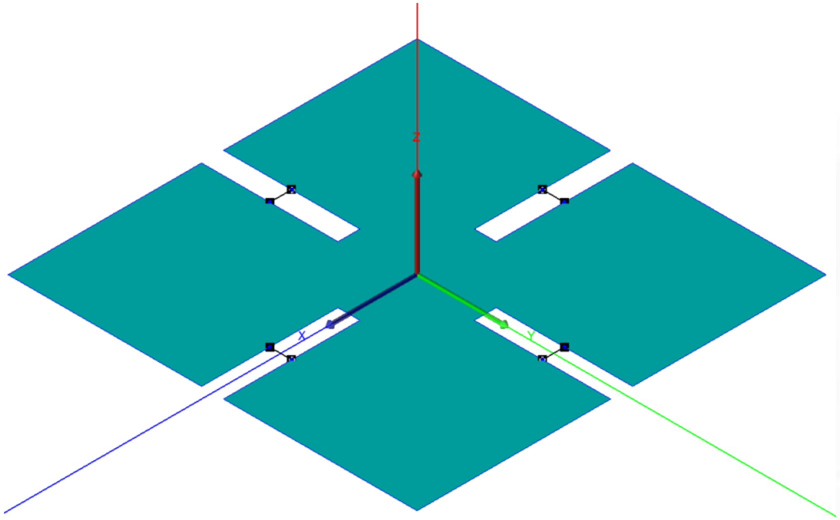


- Comparison of the radiation pattern of Mode 4 and antenna, excited to produce Mode 4



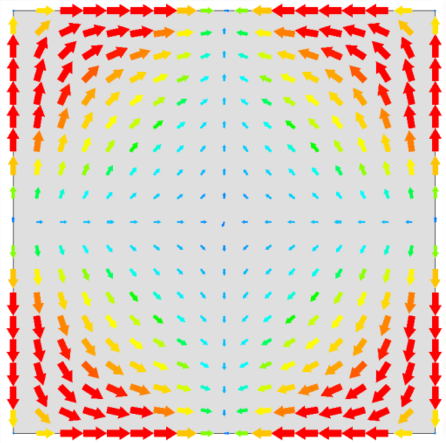
MIMO Antenna Design – Exciting of the Modes 5 and 6

- Slots created to excite modes 1, 2, 3 and 4 don't intersect current flow of modes 5 and 6
  - In order to excite modes 5 and 6, the new slots have to be created – these slots should intersect current flow of modes 5 and 6

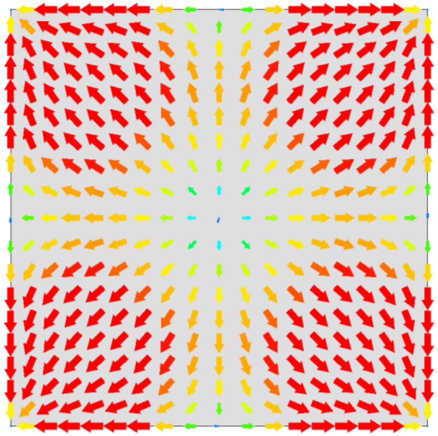


4-modes antenna
(modes 1-4)

- Currents of modes 5 and 6



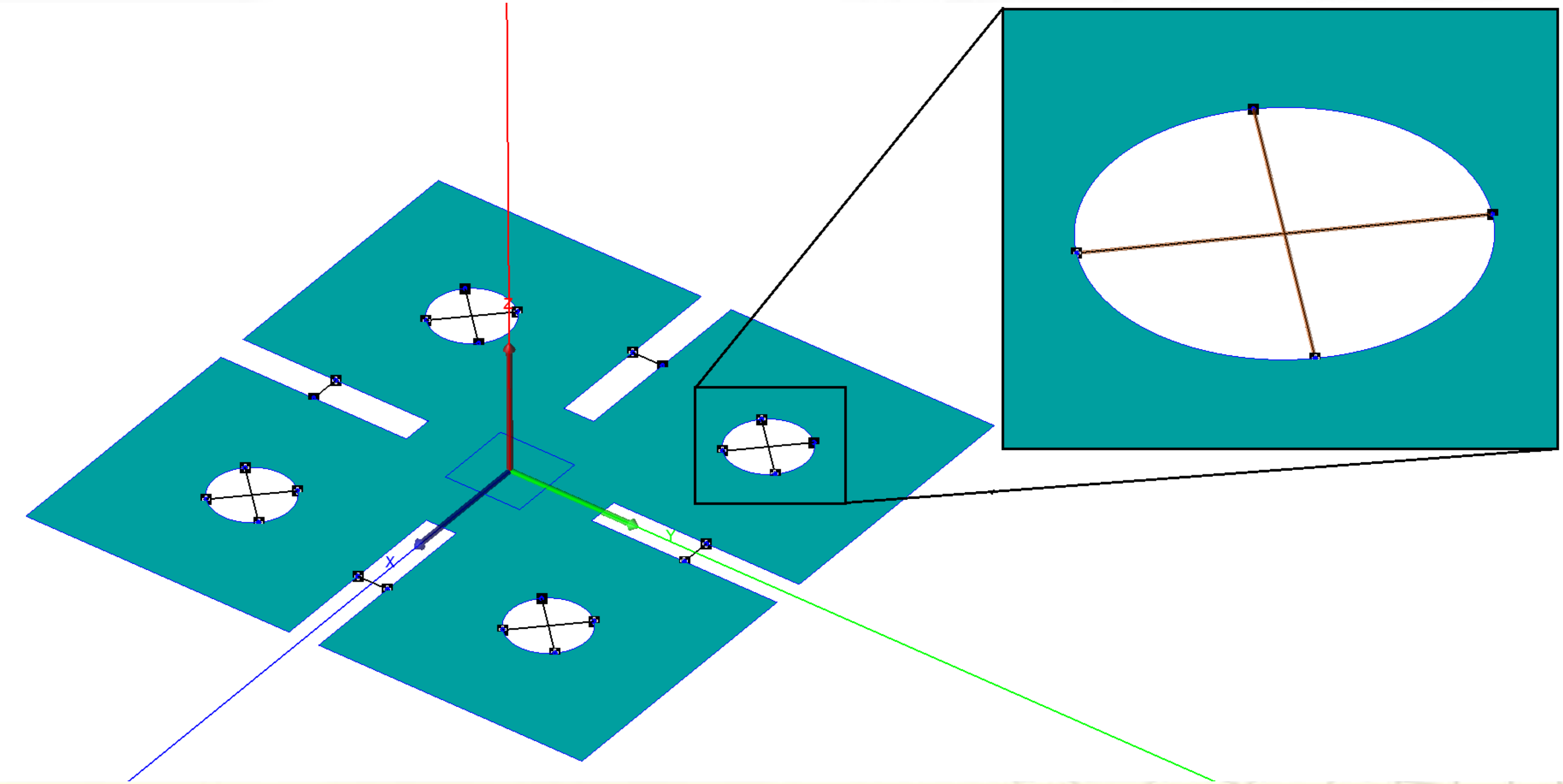
Mode 5



Mode 6

MIMO Antenna Design – Circular slots on the antenna diagonal

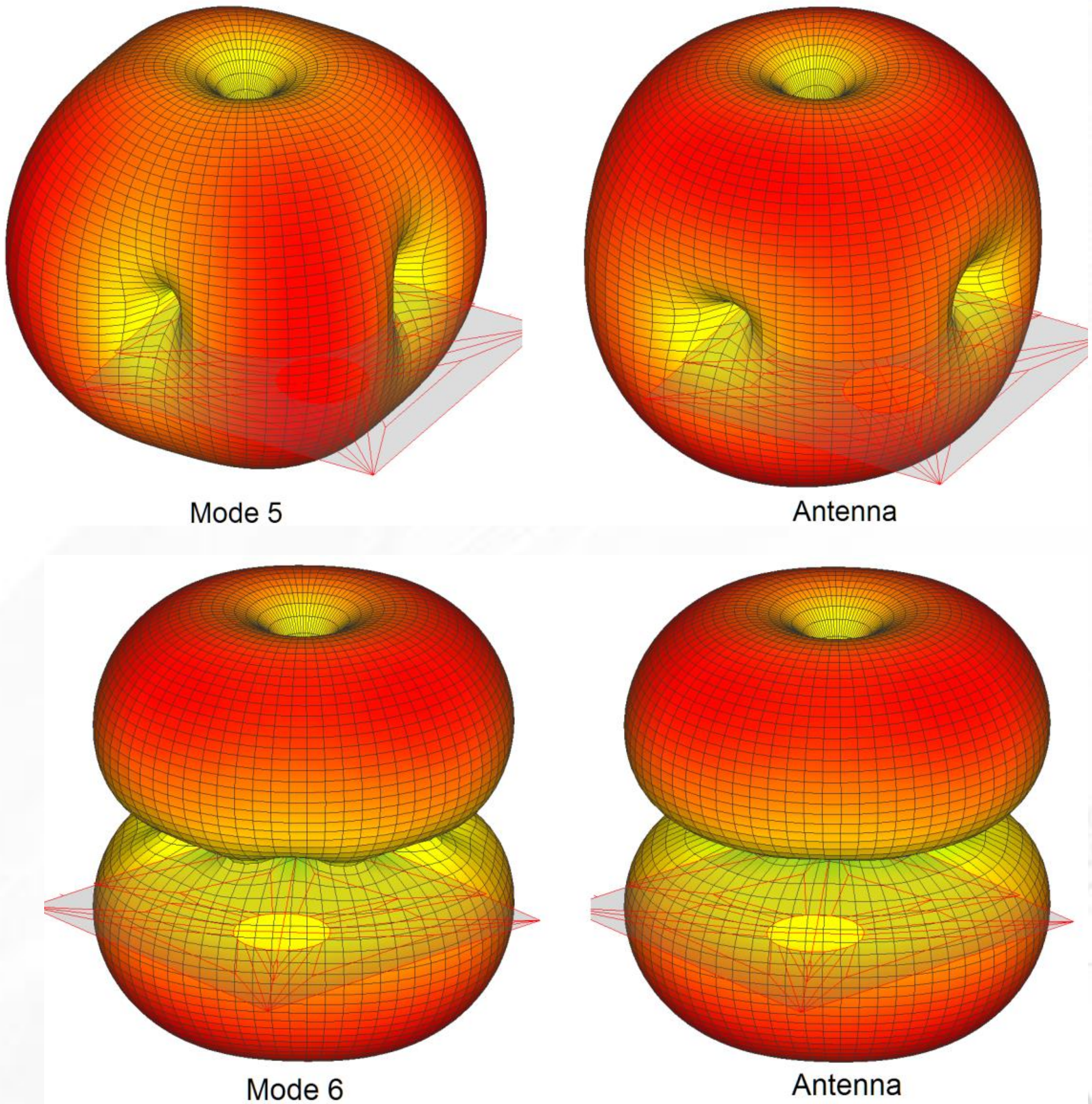
- Additional circular slots created in order to excite modes 5 and 6
  - Mode 5 excited by probes perpendicular to antenna diagonal
  - Mode 6 excited by probes parallel to antenna diagonal



MIMO Antenna Design – Radiation Patterns of modes 5 and 6

- Comparison of the radiation pattern of Mode 5 and antenna, excited to produce Mode 5

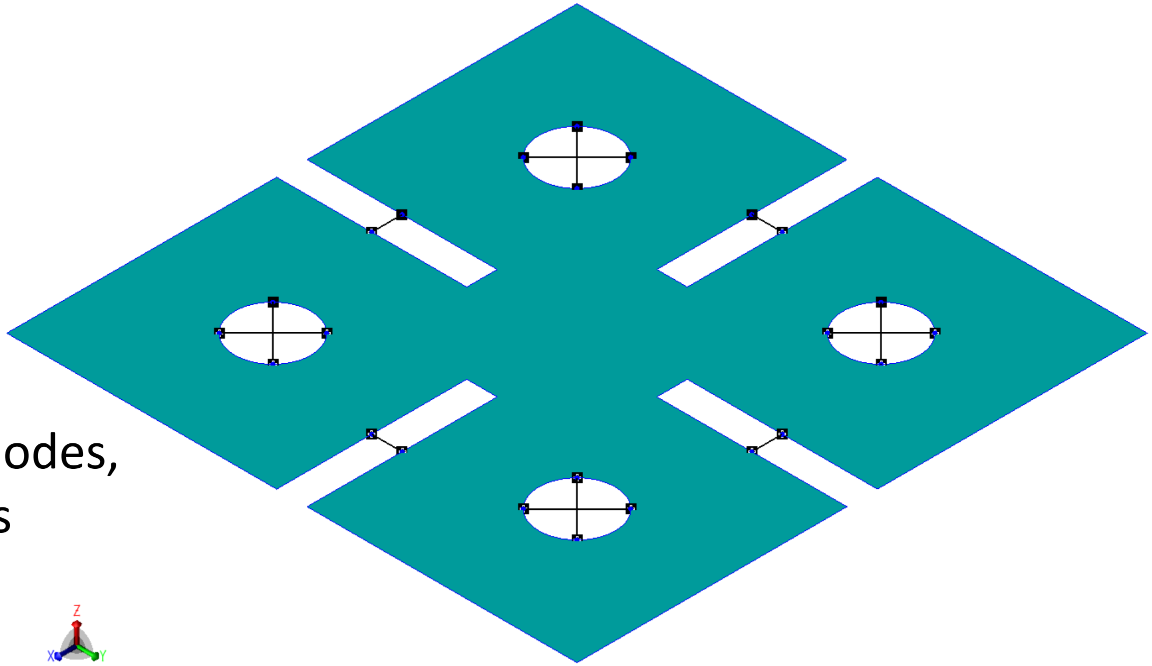
- Comparison of the radiation pattern of Mode 6 and antenna, excited to produce Mode 6



MIMO Antenna Design – Conclusion

- Starting from the results of Characteristic Modes Analysis we created 6-port MIMO antenna
  - Antenna is created by using knowledge about the natural current flows supported over the square metallic plate
  - No optimizations, systematic search, etc. have been used in the design procedure

- Each separate excitation (port) excite only one characteristic mode
  - Thanks to orthogonality between characteristic modes, coupling between ports is negligible

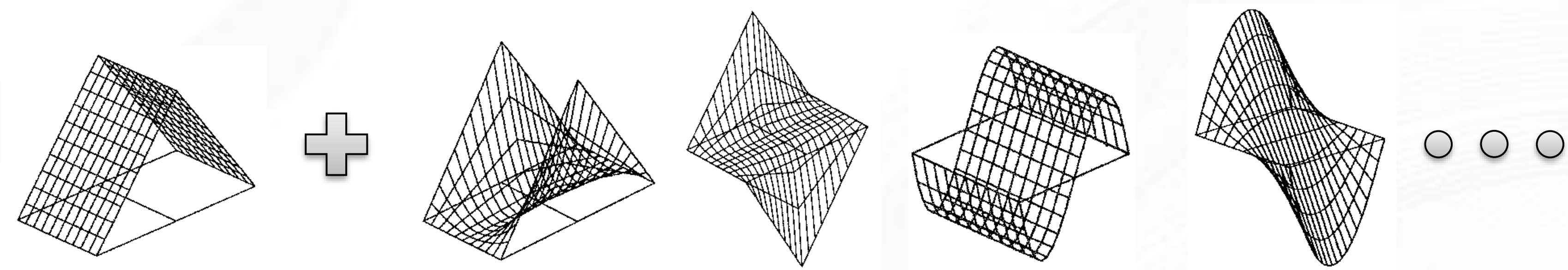


CMA for Electrically Large Structures

- Characteristic mode analysis is usually performed for problems which size do not exceeds ~10λ
  - For electrically very large objects number of modes is huge and it is very difficult to excite some of them without excited undesired modes
- Even for problems with size of several wavelengths CMA can be problematic
  - Complexity of eigenvalue calculation is $O(N^3)$
  - Large number of eigenmodes, and corresponding currents and fields, is often required for such problems
- WIPL-D is very suitable for such problems
  - Required number of unknowns is inherently reduced by using Higher Order Basis Functions and quadrilateral mesh

Basis functions used in WIPL-D

- In addition to rooftop, higher order basis functions are used



- Basis functions are polynomial functions with order up to 7th order
  - Mesh elements could be very large – up to 2λ by 2λ!
- It is possible to combine different orders of current approximation along two axes of a quadrilateral mesh element
  - Number of unknowns is minimized, even with very elongated mesh elements

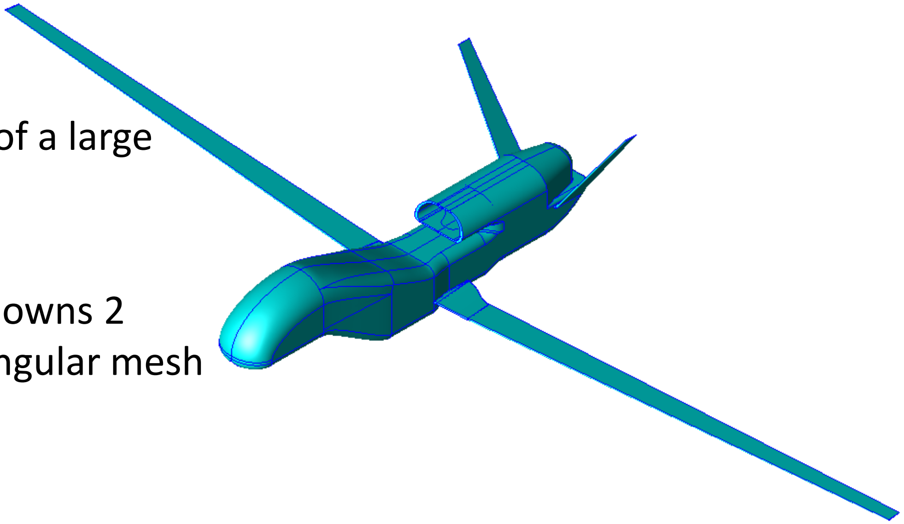
Maximum patch size 2λ by 2λ

Size of the flat plate scatterer	λ/6	3λ/6	5λ/6	7λ/6	9λ/6	11λ/6
Order of the current expansion	2	3	4	5	6	7
Total number of unknowns per patch	8	18	32	50	72	98
Unknowns per λ^2	144	48	35	30	27	25

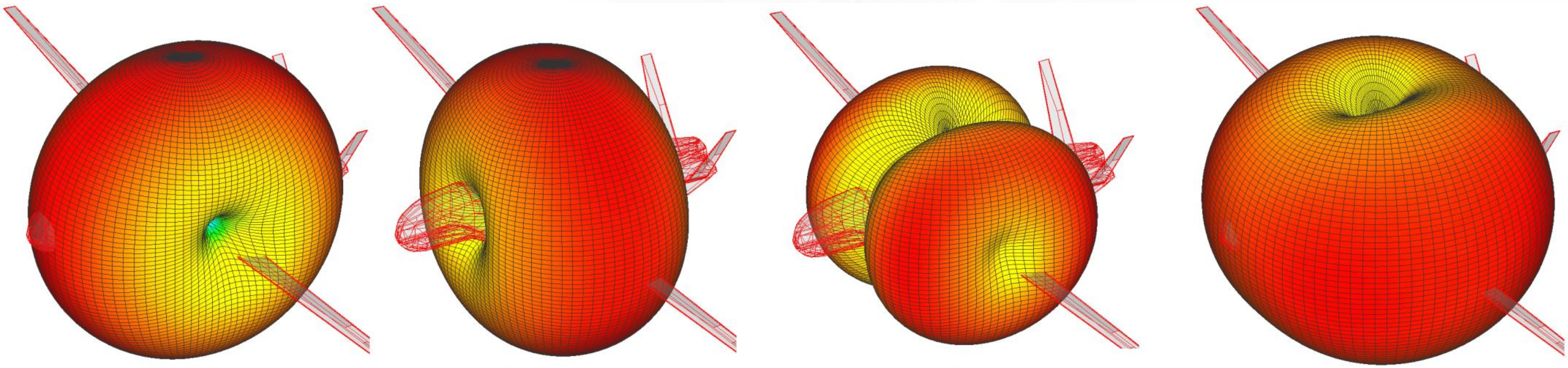
- Increase of patch size decreases the number of unknowns
  - Entire-domain expansions require 3-6 times less unknowns and 10-100 shorter analysis time than sub-domain expansion

Global Hawk at 4.5 MHz

- Airplane, global hawk, is analyzed at 4.5 MHz
  - Model is not electrically large, but it consists of a large number of elements
  - HOBFS are not used for this model
  - Usage of quad mesh reduces number of unknowns 2 times, when compare to commonly used triangular mesh
  - Number of unknowns is 2264



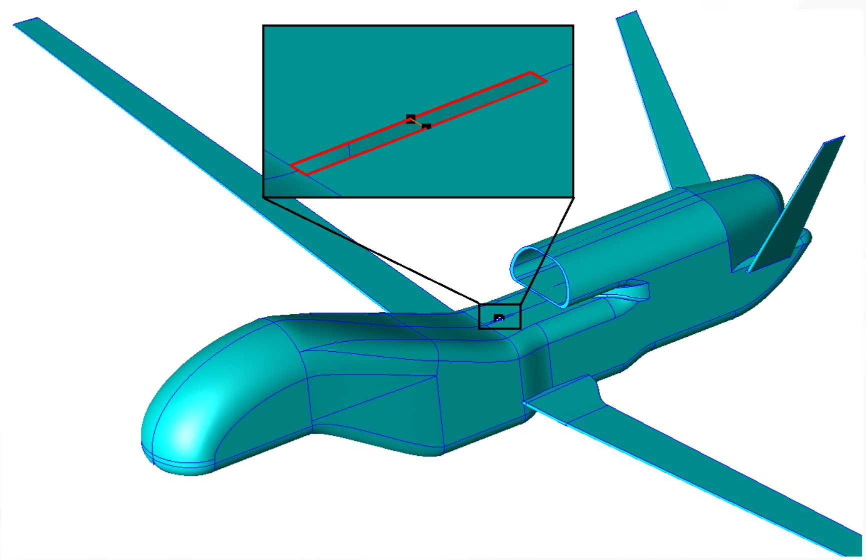
- Radiation patterns of the first 4 modes

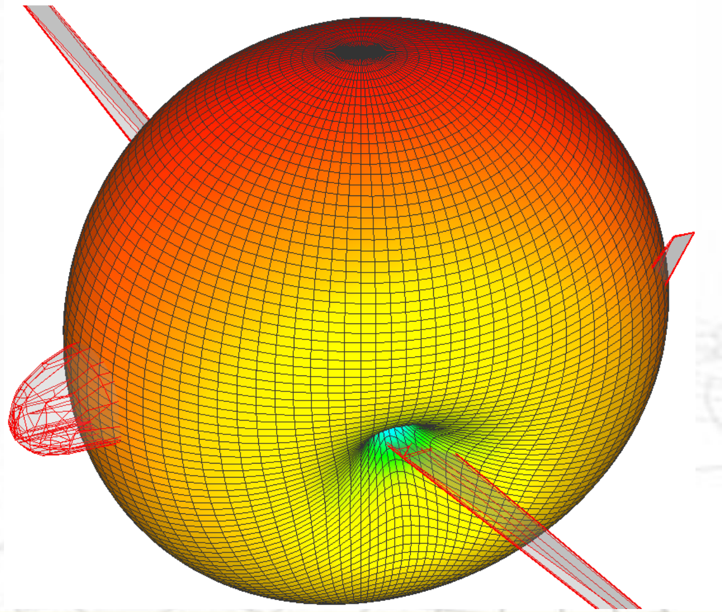


Mode 1 Mode 2 Mode 3 Mode 4

Global Hawk at 4.5 MHz – Exciting of the 1st mode

- Modal significances of the first 4 modes are: 0.93, 0.085, 0.039 and 0.0076, respectively
  - The first mode is resonant mode, while the other modes are far away from the resonance
- Physically mode 1 is half-wave dipole mode along the airplane wings
- Notch antenna is placed on the upper side of fuselage

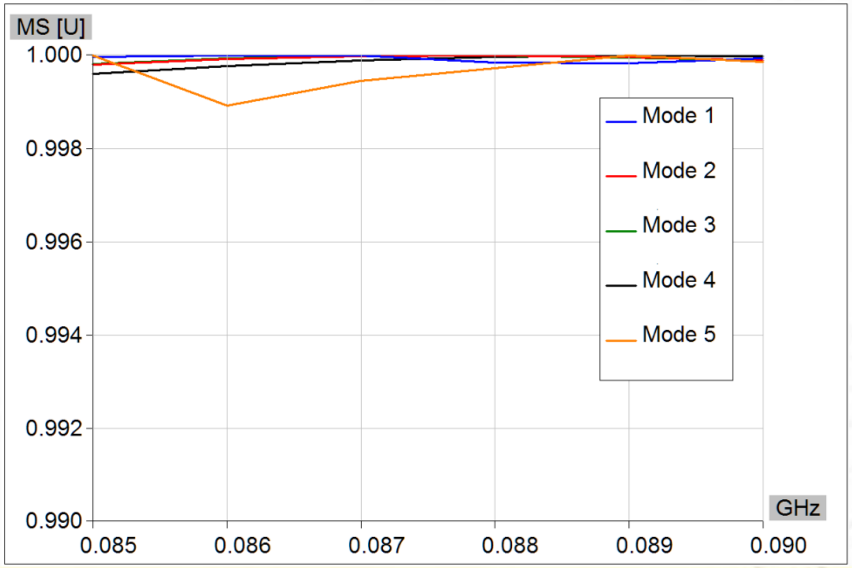




Global Hawk at 90 MHz

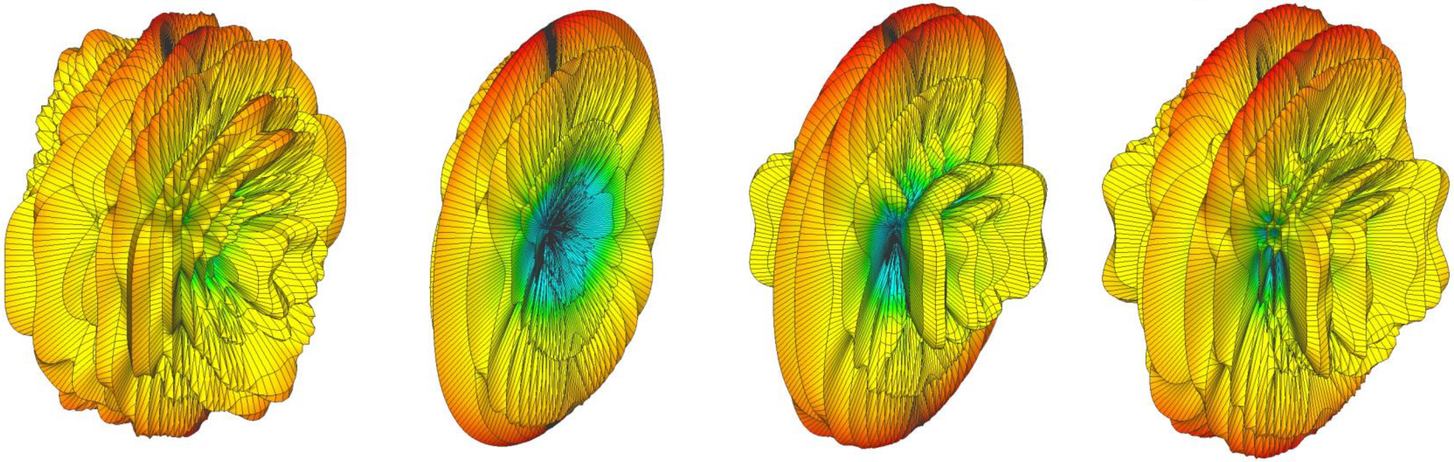
- Global hawk, is analyzed in the range between 85 MHz and 90 MHz
  - Wingspan is ~10λ
  - Number of unknowns is 6654
  - First 10 modes are calculated
- Modal Significance of the first 5 modes
  - All modes are close to resonance (with modal significance above 0.999)

  – all of them radiate very efficiently



Global Hawk at 90 MHz – Results Interpretation

- Radiation pattern of the first 4 modes



- CMA of electrically large structures in WIPL-D is not problematic at all
- Interpretation of the CMA results and its utilization in antenna design, placement, etc. is pretty difficult
  - Large number of efficiently radiating modes
  - Complex current distribution and radiation patterns
  - Difficulties in exciting of desired modes, without exciting undesired ones
  - …

Conclusion

- Characteristic Modes Analysis provide us with a deep insight into the physical behavior of the analyzed structure
  - Better understanding of the effects which exist in the analyzed structure
  - Possibility to perform antenna design starting from the knowledge about the natural current flows supported over the structure of interest
  - …
- Thanks to special techniques used in WIPL-D kernel, in order to reduce number of unknowns, CMA in WIPL-D software is very efficient

- Future work:
  - Development of CMA solver with support for dielectric/magnetic materials
  - Further efforts in order to enable better utilization od CMA of electrically large objects

Thank you!

Questions?