This paper presents the procedure for the design of Cassegrain reflector antenna. It contains theoretical consideration and foundation, as well as procedure for design in the WIPL-D software suite.

Cassegrain reflector antenna consists of two parts:

- Feeding cylindrical waveguide (feeder),
- Hyperbolic sub-reflector and main parabolic reflector (reflectors).

The design procedure is approximately divided into two steps, each corresponding to the design of single part. After a theoretical consideration, the description of the modeling in WIPL-D is provided for each step.

1. Design of Feeder

For reflector antennas, the feeding antenna is most often chosen to be horn antenna fed by waveguide. In our case we use cylindrical horn (CH) fed by cylindrical waveguide (CWG). Standard horn antenna is single mode antenna, whose beam width is not equal in E and H plane. To make it equal, we introduce a dual mode horn antenna that requires modification of the single mode horn.

The feeder design procedure consists of three phases:

- Design of CWG, according to the specified central operating frequency,
- Design of the single mode CH, and
- Design of dual mode CH.

1.1. Design of CWG

CWG is excited by a probe and typical cross-section illustration is shown in Fig. 1.

![Fig. 1. CWG Cross Section](image)

Design of CWG consists of adopting values for:

- $A_{wg}$ – waveguide radius,
- $L_{wg}$ – waveguide length (it must be large enough to allow propagation of the dominant mode only, usually half of the wavelength from probe to CWG aperture is sufficient),
- $H_{probe}$ – probe height (this should be set according to the wavelength in air or in the waveguide dielectric),
- $D_{probe}$ – probe to back wall distance (this should be set to quarter of the wavelength).

The fundamental mode propagating through a cylindrical waveguide is TE11 mode. The next mode is TM10. Cut off frequencies for TE11 and TM10 modes are:

$$F_{c}(TE_{11}) = 0.293 \frac{c}{A_{wg}}$$  \hspace{1cm} (1)

$$F_{c}(TM_{10}) = 0.383 \frac{c}{A_{wg}}$$  \hspace{1cm} (2)

where $c$ is the speed of light and $A_{wg}$ is the radius of the cylindrical waveguide. The optimum operating frequency $F_0$ is chosen to be geometrical mean value of two cut off frequencies:

$$F_0 = \sqrt{F_{c}(TE_{11})F_{c}(TM_{10})} = 0.335 \frac{c}{A_{wg}}$$  \hspace{1cm} (3)

Since $c = 10^8 \frac{m}{s}$ from (3) we obtain the radius of the cylindrical waveguide for given operating frequency:

$$A_{wg}[mm] = \frac{100}{F_0[GHz]}$$  \hspace{1cm} (4)

At the operating frequency $F_0$ the wavelength in free space $W_{L0}$ and in the waveguide $W_{Lwg}$ are given with:

$$W_{L0} = \frac{c}{F_0} \Rightarrow W_{L0} = \frac{300}{F_0[GHz]}$$  \hspace{1cm} (5)

$$W_{Lwg} = W_{L0} \sqrt{1 - \left(\frac{F_{c}(TE_{11})}{F_0}\right)^2}$$  \hspace{1cm} (6)

$$\Rightarrow W_{Lwg} = \frac{619}{F_0[GHz]}$$

Waveguide length is chosen in such a manner that higher order modes completely vanish before the aperture. The sufficient length is:

$$L_{wg}[mm] = \frac{3}{4}W_{Lwg}[mm]$$  \hspace{1cm} (7)
The distance of the probe from the back wall should be:
\[ D_{\text{probe}}[\text{mm}] = \frac{1}{4} W_{\text{LWG}}[\text{mm}] \]  
(8)

The probe length should be:
\[ H_{\text{probe}}[\text{mm}] = \frac{1}{4} W_{\text{L0}}[\text{mm}] \]  
(9)

The dimensioning of the waveguide is completed with setting the above-mentioned variables.

1.2. Design of Single Mode CH

CH is completely described with its length \( L_{\text{horn}} \) and aperture radius \( A_{\text{horn}} \). The cross section of the cone is shown in Fig. 2. The part of this cone outlined black is CH.

![Fig. 2. Cross section of CH](image)

Given the 10 dB beamwidth it is possible to determine required aperture radius as:
\[ BW = 66\frac{W_{\text{L0}}}{A_{\text{horn}}} \Rightarrow A_{\text{horn}}[\text{mm}] = 66\frac{W_{\text{L0}}[\text{mm}]}{BW} \]  
(10)

Minimum length of the cone can be obtained from:
\[ (L_{\text{cone}})_{\text{min}}[\text{mm}] = \frac{A_{\text{horn}}[\text{mm}^2]}{0.7W_{\text{L0}}[\text{mm}]} \]  
(11)

Corresponding minimum horn length can be found from:
\[ (L_{\text{horn}})_{\text{min}} = \sqrt{\left[1 - \frac{A_{\text{horn}}}{A_{\text{wg}}}\right] (L_{\text{cone}})_{\text{min}}^2} - (A_{\text{horn}} - A_{\text{wg}})^2} \]  
(12)

Actual horn length is taken to be the multiple of the minimum length:
\[ L_{\text{horn}} = C(L_{\text{horn}})_{\text{min}} \]  
(13)

where for single mode horn \( C \) should be between 1.25 and 1.5, while for dual mode it should be between 2.5 and 3.

To suppress back radiation a choke is added to the horn aperture edge. A horn with a choke looks like in Fig. 3.

![Fig. 3. Cross section of the CH with choke](image)

The choke length should be:
\[ L_{\text{choke}}[\text{mm}] = \frac{W_{\text{L0}}[\text{mm}]}{4} \]  
(14)

while the choke width \( D_{\text{choke}} \) is taken to be equal to metallic wall thickness \( T \).

1.3. Design of Dual Mode CH

The cross section of the dual mode CH is shown in Fig. 4.

![Fig. 4. Cross section of dual mode CH](image)

From the middle point of the single mode CH a CWG is formed towards the feeding CWG. This CWG guides a higher order mode and makes that two modes propagate towards the aperture of the horn. The length of this CWG can be changed with \( C_{\text{mode}} \) parameter. By changing \( C_{\text{mode}} \) it is possible to tune the radiation pattern of dual mode CH. The pattern with equal beamwidth in E and H planes is obtained if \( C_{\text{mode}} \) is chosen to be between 0.7 and 0.8.

1.4. Modeling of Feeder in WIPL-D

The complete dual mode CH feeder can be made of one BoR (body of revolution) object, two Circle objects and single wire. The cross section with the indicated objects can be seen in Fig. 5.
Since the feeder has two symmetry planes, both can be used to reduce the analysis time. There are two feeder models:

- Conical feeder 1 – the model uses one symmetry plane. Only half of the model is built, and it is suitable for calculation of both radiation pattern and input impedance.
- Conical feeder 2 – the model uses two symmetry planes. Only quarter of the model is built, and it is suitable for calculation of radiation pattern. It is not suitable for calculation of input impedance, because CWG is excited by a dipole instead of a probe. The de-embedding procedure is required to obtain the proper return loss.

In the Symbols table, we define all the design parameters mentioned in the theoretical consideration and all the relevant coordinates needed to build the model. The Symbols table can be inspected by clicking in the WIPL-D Pro.

Symbols numbered 1-6 define the design parameters of the feeder:

- $F_0$ – Reference frequency (25.5 GHz by default),
- $BW$ – 10 dB beam-width (65 deg by default),
- $T$ – Metallic wall thickness (1 mm by default),
- $Chorn$ – horn length factor (3 by default),
- $Cmode$ – dual mode horn length factor (0.71 by default),
- $Zfeed$ – Shift constant (0 by default).

Estimated minimum $N$ is given with $Nmin$, and $N$ should be even integer number larger than $Nmin$.

Symbols numbered 7-26 define auxiliary values needed for calculation. The parameter of interest is:

- $Ceq$ – Equivalent radius factor. Since the cylinder curvature is approximated by a polygon, design radius $Awg$ should be taken in the model multiplied by $Ceq$, to compensate for polygonal approximation of the cylinder curvature. $Ceq$ is chosen in such a way that cross section of cylinder (polygon) has the same surface as the circle with Radius $Awg$.

Symbols numbered 27-61 define $p$ and $z$ coordinates of the BoR object.

Modification of the models is simple. Starting from either Conical feeder 1 or 2, it is possible to obtain:

- Single mode CH - by deleting (R5,Z5), (R6,Z6), (R15,Z15) and (R16,Z16) in BoR object.
- CH without metal thickness – by deleting from (R13,Z13) to (R18,Z18) in BoR object and by completely deleting Circle 2 object.
- CH without choke – by deleting from (R9,Z9) to (R14,Z14) in BoR object.

To run simulation, click . After the simulation is done, to inspect YZS click , and for radiation pattern click .

### 2. Design of Sub-reflector and Reflector

#### 2.1. Hyperbolic Sub-reflector

The cross section of the hyperbolic sub-reflector is given in Fig. 6.

![Fig. 6. Cross section of hyperbolic sub-reflector](image)

The hyperbola is completely determined with the following parameters:

- $F_{sub}$ – focal distance of the hyperbola,
- $A_{sub}$ – Radius of the hyperbola,
**Dsub** – distance from the tip of hyperbola and coordinate axis,

**Zsub** – the height of the hyperbola edge.

**Zsub** is related to other parameters as:

\[
Z_{\text{sub}} = D_{\text{sub}} \sqrt{1 + \frac{A_{\text{sub}}^2}{F_{\text{sub}}^2 - D_{\text{sub}}^2}} \tag{15}
\]

Only the upper hyperbola is necessary for antenna design. The phase center of the dual mode CH should be placed in the Focus 2 of the hyperbola.

### 2.2. Parabolic Reflector

The cross section of the parabolic reflector is given in Fig. 7.

The parabola is completely determined with the following parameters:

**Fref** – focal distance of the parabola,

**Aref** – radius of the parabola,

**Zref** – the height of the parabola edge.

**Zref** is related to other parameters as:

\[
Z_{\text{ref}} = \frac{A_{\text{ref}}^2}{4F_{\text{ref}}} \tag{16}
\]

The Focus #1 of the hyperbola should be put in the focus of the parabola.

### 2.3. Hyperbolic Sub-reflector and Parabolic Reflector

The cross section of the reflector and sub-reflector is shown in Fig. 8.

The geometry of the reflectors is chosen to satisfy the following conditions:

- Focus #1 of the hyperbola should be placed in the focus of the parabola.
- The line which is tangential to the edge of the parabola and goes through the focus, should also be tangential to the edge of hyperbola.

### 2.4. Building Model of Reflectors in WIPL-D

The hyperbolic sub-reflector and parabolic reflector are built using predefined **Rflct** object in WIPL-D Pro.

There are two reflector and sub reflector projects:

- Cassegrain 1 (one symmetry plane used – half model) and
- Cassegrain 2 (two symmetry planes – quarter model).

Symbols numbered 1-5 are defined by the user and specify referent frequency and geometry of the reflectors:

- **F0** – Reference frequency (25.5 GHz by default),
- **Aref** – radius of the parabolic reflector (150 mm by default),
- **Dsub** – distance from the tip of hyperbola and coordinate axis,
Fref – focal distance of the parabolic reflector (112.5 mm by default),

Asub – radius of the hyperbolic sub-reflector (26 mm by default),

Fsub – focal distance of the hyperbolic sub-reflector (26 mm by default).

Symbols numbered 6-8 stand for:

Nref – Number of segments per quarter of circumference for parabolic reflector. Nref should be even integer larger than NrefE estimated in symbol #9. The estimation of NrefE is made using the criterion that maximum size of the reflector patch is no larger than 1.5 wavelength at the referent frequency F0.

Nsub – Number of segments per quarter of circumference for hyperbolic reflector. Nsub should be even integer larger than NsubE estimated in symbol #10. The estimation of NsubE is made using the criterion that maximum size of the sub-reflector patch is no larger than $\frac{\pi \lambda}{4}$ at the referent frequency F0.

BWE – Estimation of the beam-width of the feeder radiation pattern in order to have 10 dB space taper at the edge of the sub-reflector. BW from the model CH should be equal to BWE.

Symbols numbered 11-19 are additional symbols for the geometry definition and positioning, as well as auxiliary symbols.

What is left to do is to import the feeder into the project with reflector. There are two projects of the complete antenna:

- Cassegrain with conical horn 1 (one symmetry plane - half model) – suitable for calculation of both radiation pattern and input impedance.
- Cassegrain with conical horn 2 (two symmetry planes – quarter model) – suitable for calculation of radiation pattern only. The de-embedding procedure must be applied to obtain proper return loss.

To import use option Edit/Structure/Import. After the import, move the feeder phase center to hyperbola Focus #1 by specifying $Z_{feed} = Fref - 2Fsub$ in the Symbols table.

The final model with one symmetry plane (Cassegrain with conical horn 1) is shown in Fig. 9.

The Default analysis parameters are:

- Frequency range: 25.5 GHz (set in Edit/Frequency),
- Radiation pattern: $\phi=0$, 90 and -90° ≤ θ ≤ 90° at 361 points (can be modified in Edit/Output Results/Radiation)

After the analysis, the radiation patterns in 3D, E and H plane are shown in Fig. 10. The model requires only 2,792 unknowns to be simulated in WIPL-D Pro. Simulation time at the everyday inexpensive work PC (CPU: Intel i7-7700K@3.60 GHz) is under 3 seconds per frequency point.

Fig. 9. Model of hyperbolic reflector with conical horn feeder in WIPL-D Pro

Fig. 10. Radiation pattern

For the models described in this application note, please contact WIPL-D support team.