

Verifying Accuracy for Inductive Loops with Lumped Elements

Introduction

This document aims at explaining analysis of simple wire loops (inductive in their nature) in WIPL-D software package. Special attention is devoted to the case when lumped elements are also included in the circuits.

Inductive Wire Loops

In order to explain the effects of EM simulation of wire loops, we will use an example of simple wire square loop, as shown in the figure below.

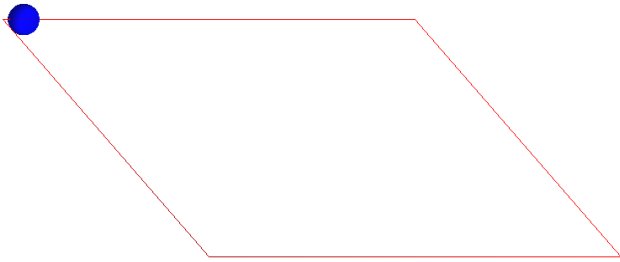


Figure 1. Square wire loop

Simulation frequency is chosen to be 15.9155 MHz which yields angular frequency of exactly:

$$\omega = 10^8 \frac{\text{rad}}{\text{s}}$$

Reactance of such loop can be considered dominantly inductive, i.e.:

$$X = \omega \cdot L$$

Consequently, multiplying the reactance by 10 one obtains inductance L in nH . Thus, for total circumference of the loop equal to 2 cm and 16 cm, WIPL-D EM simulation yields that inductance of these loops is respectively:

$$L(2) = 30.974nH$$

$$L(16) = 314.4nH$$

The accuracy of EM simulation can be verified by checking the obtained values by using a number of free calculators available on the Internet. One can be found at:

http://www.technick.net/public/code/cp_dp.php?aiocp_dp=util_inductance_square

$$L_{\text{square}} \approx N^2 \frac{2W\mu_0\mu_r}{\pi} \left[\ln\left(\frac{W}{a}\right) - 0.77401 \right]$$

Where N is number of turns, W is loop width in meters, A is loop wire radius in meters, μ_r is relative permeability of the medium. In our case:

$$R = 0.001mm, \text{ and material is set to air.}$$

This calculator gives almost the same values as WIPL-D, i.e.:

$$L_{\text{square}}(2) = 30.973nH$$

$$L_{\text{square}}(16) = 314.3nH$$

These results illustrate accuracy of WIPL-D in determining low-frequency inductance.

Loops with Lumped Elements

In addition to the inherent reactance of wire loops, lumped elements can be added to the circuit which will modify the simulation conditions. In the second set of results, lumped inductor of:

$$L = 1000nH$$

is added to the circuit on the opposite side of the loop with respect to generator, as shown in the image below.

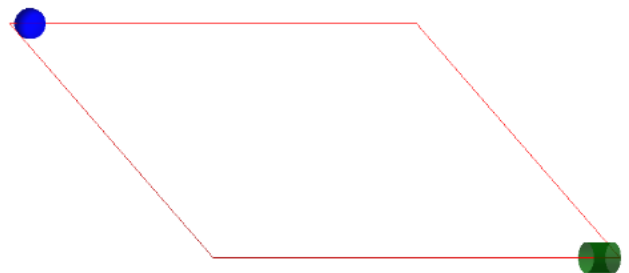


Figure 2. Loop with the lumped inductor

As the first approximation, we expect that total inductance of such "mixed" loop is equal to:

$$L_m(2) = L + L(2)$$

$$L_m(16) = L + L(16)$$

WIPL-D simulation yields:

$$L_m(2) = 1031.3nH$$

$$L_m(16) = 1317.4nH$$

which is almost exact to what we expected.

If the simulation is run in the frequency range from 1 MHz to 100 MHz, we obtain the following results for loop impedance (imaginary part).

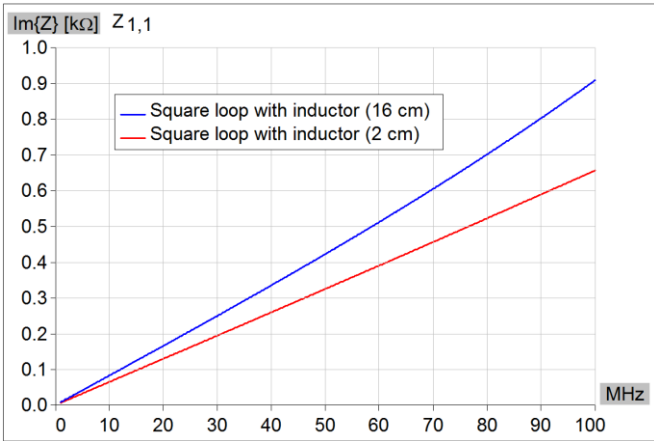


Figure 3. Loop with the lumped inductor of 1000 nH

The results differ in ratio of:

$$\frac{L_m(16)}{L_m(2)} = 1.277$$

i.e. for 27.7% in the entire frequency range.

It should be noted that by decreasing this ratio tends to:

$$\frac{L_m(16)}{L_m(2)} = 10.15$$

i.e. 915%.

In the circuit theory, if the lumped inductor value is increased well beyond 1000nH, the loop impedance becomes negligible and the ratio should tend to 1. However, in the 3D simulation circuit, another effect becomes dominant, the resonant frequency. If the lumped inductor is set to:

$$L = 100 \mu H$$

the EM simulations results are shown in frequency range from 1 to 100 MHz in the figure below.

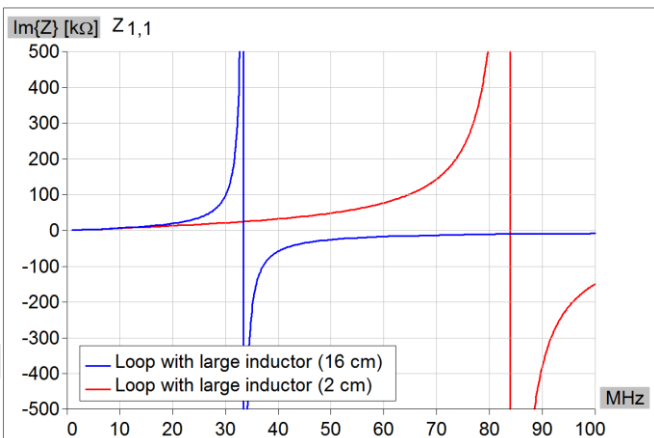


Figure 4. Loop with the lumped inductor of 100 μH

It can be observed that resonance frequency is halved when we compare larger and smaller loop.

This can be proved by the following simple model. Namely, in the first approximation the loop can be represented as transmission line of certain characteristic impedance Z_c , ended by lumped inductor L . The impedance seen from generator is:

$$Z = Z_c \frac{j\omega L + jZ_c \cdot \text{tg}(\beta l)}{Z_c + j \cdot (j\omega L) \text{tg}(\beta l)}$$

where phase coefficient β can be written as:

$$\beta = \frac{\omega}{c}$$

c is velocity of light and l is length of the line. Note that this transmission line is electrically short, so we can assume that:

$$\text{tg}(\beta l) \approx \beta l$$

The denominator can be thus considered as:

$$Z_c - \omega^2 L \cdot \frac{l}{Z_c \cdot c}$$

Having in mind that characteristic impedance can be expressed in terms of capacitance per unit length, i.e.:

$$Z_c = \frac{1}{c \cdot C'}$$

the denominator can be simplified to

$$Z_c - \omega^2 LC' \cdot l$$

Since, L and C' are same for both loop dimensions, it is obvious that by increasing l four times the resonant frequency is decreased 2 times, exactly as shown in the Fig. 4.